NAVAER

INVESTIGATION ON THE POSSIBILITY OF ESTABLISHING A SPACE SHIP IN AN ORBIT ABOVE THE SURFACE OF THE EARTH

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INVESTIGATION ON THE POSSIBILITY OF ESTABLISHING A SPACE SHIP IN AN ORBIT ABOVE THE SURFACE OF THE EARTH

In view of the recent progress in the field of rocket missiles it may prove advantageous to review the possibility of establishing a space ship in an orbit above the surface of the earth.

At present very little is known concerning the exact nature of the space above an altitude of fifty miles. More detailed information which could be obtained by a ship of this type would enhance our knowledge in the fields of science, communication and meterology. It is also a basic step in the realm of space travel. From a more immediate point of view the ship may be employed to obtain information of military importance, to direct or convey explosive charges or to intercept and combat enemy craft of a similar nature.

The mission for which the ship is employed would determine the desirable orbit in which it should travel. For carrying explosive charges or for reconnaissance of enemy positions it would be desirable to travel an orbit just above the atmosphere possibly at an elevation of about sixty miles. Under such conditions all the necessary information could be obtained in a few trips over the target, and it would be of little consequence if the ship is downed due to air resistance after the first few hours of service. The use of wings for landing such a craft may be desirable, although television or automatic photography could supply the desired information, without personnel.

A more permanent orbit may be established at a higher altitude. It is estimated that above five hundred miles the resistance of the air would do little to slow the ship, and in such an orbit the ship would remain indefinitely. This orbit may prove more desirable for communications or for scientific observations. Of special interest is a circular orbit at 22,300 miles above the surface of the earth, where the ship would make one revolution

Summary:

A theoretical analysis and check point calculations have been made to determine the possibility of establishing a ship in a space orbit above the surface of the earth. Theoretical curves have been prepared which show the minimum energy necessary for a circular orbit at any elevation above the earth. Curves showing the minimum mass ratio required to develop this energy with fuels of a given specific impulse are presented. An attempt has been made to ascertain whether a rocket can be constructed with the required mass ratio. To achieve this end a formula for the weights of the essential structure for a single step rocket was developed. This formula was modified to cover weight relations in multistage rocket construction.

Sample calculations have been made for a particular orbit, employing available fuels, both for a single stage and a multi-stage rocket ship. The calculations indicated that a single stage rocket would not suffice to place the ship in this orbit. However, the orbit may be reached with a multistage design, even with alcohol and oxygen as the fuel. The initial mass required to place a unit mass in the orbit is given.

Conclusions:

In making the sample calculations assumptions have been made concerning the orbit, the path to the orbit, the specific impulse of the fuel, and the weight of the structure. It is quite possible that a more conservative path to the orbit may be obtained for a useful project. However, every effort has been made to reduce the weight factor and the figures given may prove to be optimistic. It is certainly true that considerable effort will be necessary in the design of a space ship which will meet these weight and structural requirements. Considerable research must also be done on fuels and on motor design.

In view of the numerous advantages which may be obtained from such a device and the extensive field of possibilities which a successful project of this type may initiate, it is felt that a more exacting study of the possibilities should be inaugurated. It is recommended that a detailed structural design be made to ascertain the accurate weights of a space ship and all components.

It is further recommended that the research program on rocket fuels and motors be intensified in an attempt to obtain fuels with a higher specific impulse and to develop large motors which will operate with these fuels for the required time.

The calculations of the next few paragraphs treat the possibility of placing an object in a circular orbit. In the consideration of the problem, it is deemed important to determine the requirements under the optimum condition of operations. The minimum amount of energy required is determined, then the possibility of supplying this energy is treated. First, the required mass ratio is determined in terms of heat content of the fuel under the assumption of a machine which transforms 100% of the fuel energy into the energy of the space ship. Second, the required mass ratio is determined for a rocket operating in a conservative field, e.g., air resistance is neglected. (These theoretical studies should be followed by a study of the possibilities of future fuels; however, the recent report by the Gilliland Committee is sufficient for the present.) Third, the maximum mass ratio for a rocket is computed based upon an estimation of the weight of the essential structure. Lastly, the study includes an estimate of the possibilities based upon present tested materials.

<u>Notation</u>:

m = mass of space ship

M = mass of the earth

κ^2 = the proportionality constant

R = radius of the space ship orbit

ω = angular velocity of space ship

R_o = radius of the earth - miles

ω_o = angular velocity of the earth - radians

y = ratio of initial mass to final mass = <u>mass ratio</u>

x = B.T.U. per pound of the sum of oxidizer and fuel

$\Delta(\ )$ = increment of ( )

c = effective jet velocity - feet per second

V = velocity of space ship - miles per hour

v = velocity of space ship - feet per second

I = specific impulse of fuel

W = weight of non-pay load, less fuel

Notation: (cont'd)

d = average relative density of the propellant

D = maximum diameter

L = overall length

D/L = .1175 fineness ratio

h = altitude above earth - miles

1. Total Energy for a Space Ship

The minimum amount of energy required to establish a space ship neglecting air resistance, is equal to the sum of the change in kinetic energy and the change in potential energy of a mass at rest on the surfact of the same mass in a space orbit. In order for a body to remain at a given altitude in a circular orbit about the earth, the effects of the heavenly bodies neglected, the centrifugal force must equal the gravitational force.

Thus,

(1) $m\omega^2 R = k^2 \frac{mM}{R^2}$

Now,

(2) $V^2 = \omega^2 R^2$, or from (1)

$\omega^2 R^2 = \kappa^2 \frac{M}{R}$

Thus,

(3) change kinetic energy $= \Delta KE = \frac{m}{2} \left(\frac{\kappa^2 M}{R} - \omega_o^2 R_o^2 \right)$

For vertical motion

$$m \frac{d^2 R}{dt^2} = - \frac{\kappa^2 M}{R^2} m$$

whence,

$$\frac{dR}{dt}^2 = 2 \frac{\kappa^2 M}{R}$$

Therefore,

(4) change potential energy $= \Delta P.E. = \kappa^2 M \left(\frac{1}{R_o} - \frac{1}{R} \right) m$

The sum of the change in potential and kinetic energies given the total energy change.

Hence, from (3) and (4)

(5) Δ total energy $= m \left[\kappa^2 M \left(\frac{1}{R_o} - \frac{1}{2R} \right) - \frac{\omega_o{}^2 R_o{}^2}{2} \right]$

See Graph 1.

2. Revolutions Per Day

The velocity of the space ship varies with altitude. It is interesting to know the revolutions per day for a given altitude.

(6) Revolutions per day $= \frac{V\ (24)}{2\pi R}$

$= \frac{4.265 \times 10^6}{R^{3/2}}$, from (2)

See Graph 2.

3. Minimum Mass Ratio for 100% Energy Transfer

If 100% of the heat energy of the fuel is transferred to the energy of the final mass (none lost to lifting or accelerating fuel mass) then the mass ratio required may be expressed in terms of the thermal units of the fuel.

(7) $8.66 \times 10^5 (1 - \frac{1981.7}{R}) = (y - 1)x$

This relation is obtained by equating the total energy required (5) to the heat energy of the fuel. The pounds of fuel $= m(y-1)$.

See Graph 3. Points are given indicating the values for alcohol and liquid oxygen, gasoline-liquid oxygen, and liquie hydrogen.

The conditions are ultra-ultra and cannot be obtained in any machine. However, these simple considerations show that a single stage rocket will not suffice unless it is possible to construct one with a more favorable mass ratio than the V-2. The mass ratio for the V-2 was 3.175, whereas the minimum required by the fuels considered are 4.6, 4.4 and 3.7, respectively.

4. Minimum Mass Ratio for Rocket

The rocket is a possible machine for producing the energy transfer. Here it is assumed the rocket in space is not affected by the gravity or air resistance, then for a coordinate system fixed in this space

$$m \frac{dv}{dt} = c \frac{dm}{dt},$$

where c, the effective exhaust velocity of the rocket, is assumed to be constant.

Integration yields

$$V = V_o + c \log \frac{m_o}{m} \tag{8}$$

$$V = V_o + c \log y$$

Thus, for a given mass ratio y,

$$\Delta V = c \log y$$

Here the rocket is operating in a conservative field, so the maximum velocity is a measure of the maximum transfer of energy which can be attained. Thus, the smallest mass ratio to supply the energy to the space ship by a rocket would be given by equating the maximum velocity attainable by the rocket to the velocity equivalent required to give the minimum total energy need for the space ship. Under the above assumption, where the energy of the rocket is not utilized to overcome air resistance or the potential energy change of the burned fuel $(m_o - m)$, the energy the rocket could impart to the final mass m is

$$\frac{m}{2} \left[(\omega_o R_o + c \log y)^2 - (\omega_o R_o)^2 \right]$$

Thus, the minimum mass ratio for a rocket is found by equating this value to the change of energy required in equation (5). Namely:

$$\frac{(\omega_o R_o + c \log y)^2 - (\omega_o R_o)^2}{2} = \kappa^2 M \left(\frac{1}{R_o} - \frac{1}{2R}\right). \tag{9}$$

When c is replaced by its equivalent specific impulse $c = 32.2\ I$, the relation becomes

$$(10)\quad I = \frac{1}{32.2 \log y}\left[\sqrt{2\kappa^2 M\left(\frac{1}{R_o} - \frac{1}{2R}\right)} - \omega_o R_o\right].$$

Graph 4 shows this minimum mass ratio to place a mass m in a space orbit as a function of I. Here, nothing has been dropped. The mass m includes the hull, motors, fuel pumps and other materials not used as fuel for propulsion. Such a system is called a single step rocket. Naturally, a construction which would drop materials when they lose their usefulness would be better.

5. Maximum Mass Ratio for Rocket

The question of the possibility of establishing a space ship by means of a rocket cannot be answered without an estimate of the weights of essential construction parts; framework, motors, fuel pumps, etc. This is necessary regardless of the number of stages of rockets. The exact weight cannot be determined without a detailed design. When one detailed design has been made, then the weights of structures for various size and ships may be determined by dimensional analysis. Available empirical dimensional analysis for aircraft structures have been used as a guide for the weight estimates of similar rocket parts. Other weights have been computed based upon the stresses involved, using present available metals.

From a preliminary design the weight in pounds of the non-pay load, excluding fuel were estimated to equal

$$(11)\quad W = \overset{\text{shell}}{1.93\,(LD^2)^{.85}} + \overset{\text{fuel tanks}}{.005\ (\text{weight of fuel})} + \overset{\text{motor + fuel pump}}{76\left\{1 = \frac{1}{4\sqrt{d}}\right\}\frac{(\text{Thrust})}{c}}$$

$$+\overset{\text{fuel for fuel pumps}}{.017\ (\text{weight of fuel})} + \overset{\text{control fins}}{.25\ (DL)^{1.25}} + \overset{\text{internal fixed controls}}{600}$$

This relation enables one to determine the ratio of the weights of the necessary auxiliary parts to the total weight.

Under the hypothesis that the formula is correct, the ratio gives maximum mass ratio for a single stage rocket.

Graph 4 shows the minimum mass ratio required to place a ship in a space orbit for a specific fuel. If this is greater than the maximum mass ratio as determined by the weight formula, then a space ship cannot be established by a single stage rocket employing this particular fuel. For a given acceleration and its duration the mass ratio determined by the weight formula is a function of the density and the specific impulse of the fuel. The density affects the overall size of the rocket, the specific impulse affects the size of the motor, and both affect the weight of the fuel pumps. Of the present tested fuels, liquid hydrogen and liquid oxygen appear to be the best. Graph 5 given the maximum mass ratio for a rocket using this fuel. (Note the pay load has not been included.) For extremely large weights the maximum mass ratio possible to construct exceeds the minimum mass ratio required by Graph 4. However, when the losses due to air resistance and the deviation from the best path are included, the mass ratio required for the above design approaches 12.11. If a more effective control system were employed so as to reduce the required value of 12.11, it is thought that the overall weight would be more. More detailed study would be necessary to obtain the optimum condition. (See Appendix A.)

This analysis indicates that the establishment of a space ship by a single stage rocket operating on the present tested fuels is doubtful. Improvements of fuels and metals may change the picture.

6. N-Stage Rockets

Again assume that a formula for the weights may be computed, where all stages after the first are similar. The first stage, which operates in the atmosphere, must be streamlined so as to minimize the drag and must have external control fins, which are not necessary for the remaining steps. The rockets are to be telescoped inside one another. With H_2 and O_2 fuel, Graph 5a gives the assumed mass ratio possible for the first stage, while Graph 5b gives the ratios for the remaining stages. Graph 5b is developed from the formula (11), where the term for control fins has been deleted. It has been found by numerical integration, using drag data based upon the V-2, that the rocket may be propelled vertically through the atmosphere with a mass ratio 1,606:1. (See Appendix A.) The velocity required to establish a space ship in a circular orbit about the earth starting from a point just outside the atmosphere is given by Graph 7. The following analysis shows that a space ship can be established by a step rocket.

Assume an orbit 1,000 miles above the earth surface and a final pay load of 5000 pounds. The velocity equivalent to the energy required to establish the space ship from above the atmosphere (350,000 feet above sea level) is 26,750 feet per second.

Consider the steps in reverse

n^{th} step

5,000	pounds pay load
3,200	pounds essential structure
11,800	pounds fuel
20,000	pounds total weight

2.44:1 Mass Ratio

From Graph 8, this rocket would add a velocity of

.88 c = (88) (14,000) = 12,320 feet per second

$(n - 1)^{st}$ step

20,000	pounds pay load
11,000	pounds essential structure
57,000	pounds fuel
88,000	pounds total weight

2.84:1 Mass Ratio

From Graph 8, this rocket would give from rest a velocity of

1.15 c = (1.03) (14,000) = 14,600 feet per second

Thus, the two steps will give a velocity of 26,000 feet per second. This is slightly greater than the necessary amount required to take a body above the atmosphere and place it in the space orbit. A mass ratio of 1.605 is required to pass through the atmosphere vertically with fuel H_2 and O_2. Thus, for the first step

88,000	pounds pay load
21,750	pounds essential structure
64,250	pounds fuel
174,000	pounds initial total weight

1.605:1 Mass Ratio

The three-step rocket outlined above would establish a space ship of 5,000 pounds of pay load (3,200 pounds essential structure remains with the 5,000 pounds, leaving total weight 8,200 pounds). The overall mass ratio is 35:1.

The above example assumes a hydrogen-oxygen motor which runs continuously producing a jet velocity of 14,000 feet per second in a vacuum. Since this has not been experimentally verified, it is of interest to consider the possibility of establishing the space ship with the fuel alcohol and liquid oxygen.

Similar assumptions for the shape and weights admit the following:

5,000	pounds pay load
5,000	pounds structure
25,000	pounds fuel
35,000	pounds total

315:1 Mass Ratio

Increment of velocity attained by n^{th} step 1.25 c = 11,740 feet per second.

$n - 1^{st}$ step

35,000	pounds pay load
35,000	pounds structure
280,000	pounds fuel
350,000	pounds total

5:1 Mass Ratio

Increase in velocity during (n - 1^{st}) step = 1.60 c = 15,000 feet per second. The velocity required is 26,750 feet after the missile is through the atmosphere. The above two steps will give a velocity slightly greater than that required to place an object above the atmosphere in the circular orbit.

First step

350,000	pounds pay load
110,000	pounds structure
506,000	pounds fuel
966,000	pounds total initial weight

2.1:1 Mass Ratio

This gives sufficient energy to go through the atmosphere. Hence, under the above assumption a space ship may be established by the present known tested fuel, but the ratio of initial weight to pay load is about 19 :1. This could possibly be improved by more steps. However, the best ratio which could be attained ideally by continuous dropping of weight is 25:1, with alcohol and oxygen.

It is assumed in both of these examples that the path after leaving the atmosphere is an ellipse, that the motors are shut off as the velocity required to take the object (ship) out to 1,000 miles above the earth is attained and the motors are turned on again to increase the velocity enough so that the ship will change to a circular orbit.

Before leaving the subject it is well to compare the above statements with an existing rocket. The following values are given for the V-2.

750	Kg pay load
4000	Kg weight emply
12700	Kg weight full

These values give a much higher percentage of weight for essential structure. Here the percentage is 25.6 as compared to the above estimates of about 12.5 by the above weight formula. This difference makes a tremendous difference in the overall mass ratio. Assuming the weight hold for all stages, the initial weight to the final pay load is 7,800:1.

APPENDIX A

The mass ratio has been calculated, step by step method, for a simple path from the surface of the earth to the desired orbit. This path consists of a vertical rise through the atmosphere and an elliptical path normal to the radius at the end of the vertical rise and tangent to the final circular path.

The path requires that the ship be turned through 90° at the end of the vertical rise, and again through 180° before applying the final acceleration in the circular orbit.

Approximations indicate that it is possible to achieve greater efficiency by going through the atmosphere at an inclined angle, or by turning the ship before leaving the atmosphere. However, the exact calculations for such a path require data on drag coefficients and control equipment (complication of weight) which is not readily available. Such calculations are more complicated and are not considered within the scope of this report.

Care has been taken to insure that the ship has the correct angle in the elliptical path when it has acquired the correct velocity.

Data for atmospheric density and pressure was taken from the Standard Atmosphere Tables for values up to 50,000 feet, and from data supplied by the Bureau of Standards for higher altitudes.

It has been assumed that the ship would be designed along lines similar to the German A-4 rocket, and data for the drag coefficient has been taken from A-4 curves.

The variation of exhaust velocity with pressure has been taken from curves in the text for the Jet Propulsion course given at California Institute of Technology for Officers of the Army and Navy.

The table below gives the energy per unit mass that is transmitted to the rocket while accelerating vertically through the atmosphere. The data is based on a space ship which is fifteen feet in diameter and has an original weight of 200,000 pounds. The ship starts at an elevator of 14,000 feet above sea level.

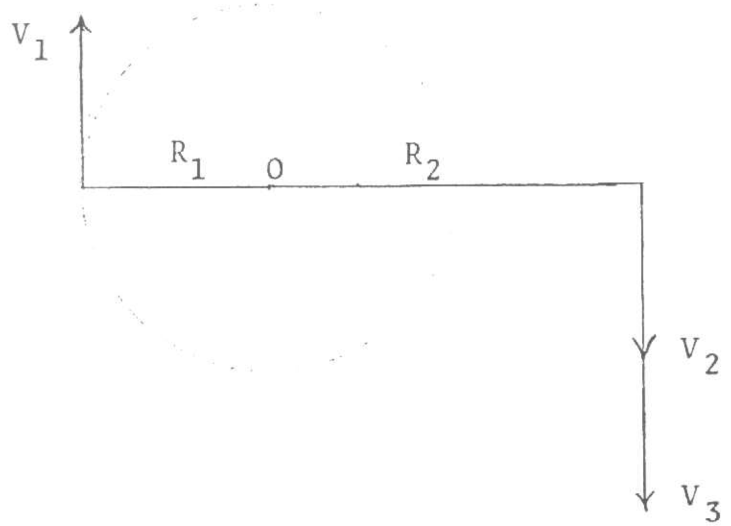
$$\frac{E}{M} = \frac{1}{2}V^2 + gh$$

Net Accel.	time ()	V	h	M	½V^2	gh	E/M
1g	12	411	2415	186707	84500	77800	162300
2g	8	537	2111	186707	144200	68000	212200
3g	6	586	1712	186707	171800	55150	226950
2g	16	1136	8770	173412	645000	283000	928000
3g	12	1239	7171	173412	767000	231000	998000
2g	24	1780	20371	160118	1584000	656000	2240000
3g	18	1936	16644	160118	1874000	536000	2410000

The following tabulation shows the mass ratio required to go through the atmosphere (to an elevation of 350,000 feet) with constant rate of mass dissipation and with a maximum acceleration of six times gravity.

Fuel	C*	Initial Accel. (Net)	Final Accel. (Net)	Time of Accel.	Mass Ratio
Alcohol - Oxygen	5350	2g	6g	42 sec	2.100
Hydrogen - Oxygen	8020	3g	6g		1.606

The mass ratio required to take the ship from a point outside the atmosphere to an orbit at 1000 miles elevation is calculated as follows:



$R_1 = 20.89 \times 10^6 + 350{,}000 = 21.24 \times 10^6$ feet

$R_2 = 20.89 \times 10^6 + 5.28 \times 10^6 = 26.17 \times 10^6$ feet

$a = \frac{R_1 + R_2}{2} = 23.70 \times 10^6$ feet Semi major axis of ellipse

$2a = 47.41 \times 10^6$ feet

$V_1^{\,2} = 2K^{2M} \frac{10^{-6}}{21.24} - \frac{10^{-6}}{47.41}$ (Celestial Mechanics, Moulton, Page 150)

$= 2 \times 1.412\ (0.02109) \times 10^{10} = 7.330 \times 10^{10}$

$V_1 = 27080$ feet per second. Initial velocity in the ellipse

$V_2^{\,2} = 2 \times 1.412\ (0.03823 - 0.02109)\ 10^{10} = 4.840 \times 10^{10}$

$V_2 = 22{,}000$ feet per second. Final velocity in the ellipse

$V_3 = 23{,}200$ feet per second. Velocity in circular orbit of radius R_2

$V_3 = V_2 = 1200$

The velocity necessary to establish the circular orbit from a point above the atmosphere is

$V = 27080 + 1200 - 1530 = 26750$

Equating this value to that which may be produced by a rocket, gives

$c \log \frac{M_2}{M} = 26750$

Table 3 shows the mass ratio required to place a ship in the orbit at 1000 miles elevation above the surface of the earth. This is a single stage ship which does not exceed 6g acceleration and starts at an elevation of 14,000 feet.

Fuel	Mass Ratio in Air	Exhaust Velocity in Vacuum	Mass Ratio in Ellipse	Total Mass Ratio
Alcohol - oxygen	2.100	9310	17.70	37.38
Hydrogen - oxygen	1.606	14000	6.77	

More favorable mass ratio figures will result if we are able to reduce the mass of the ship at a point or points prior to the final acceleration.

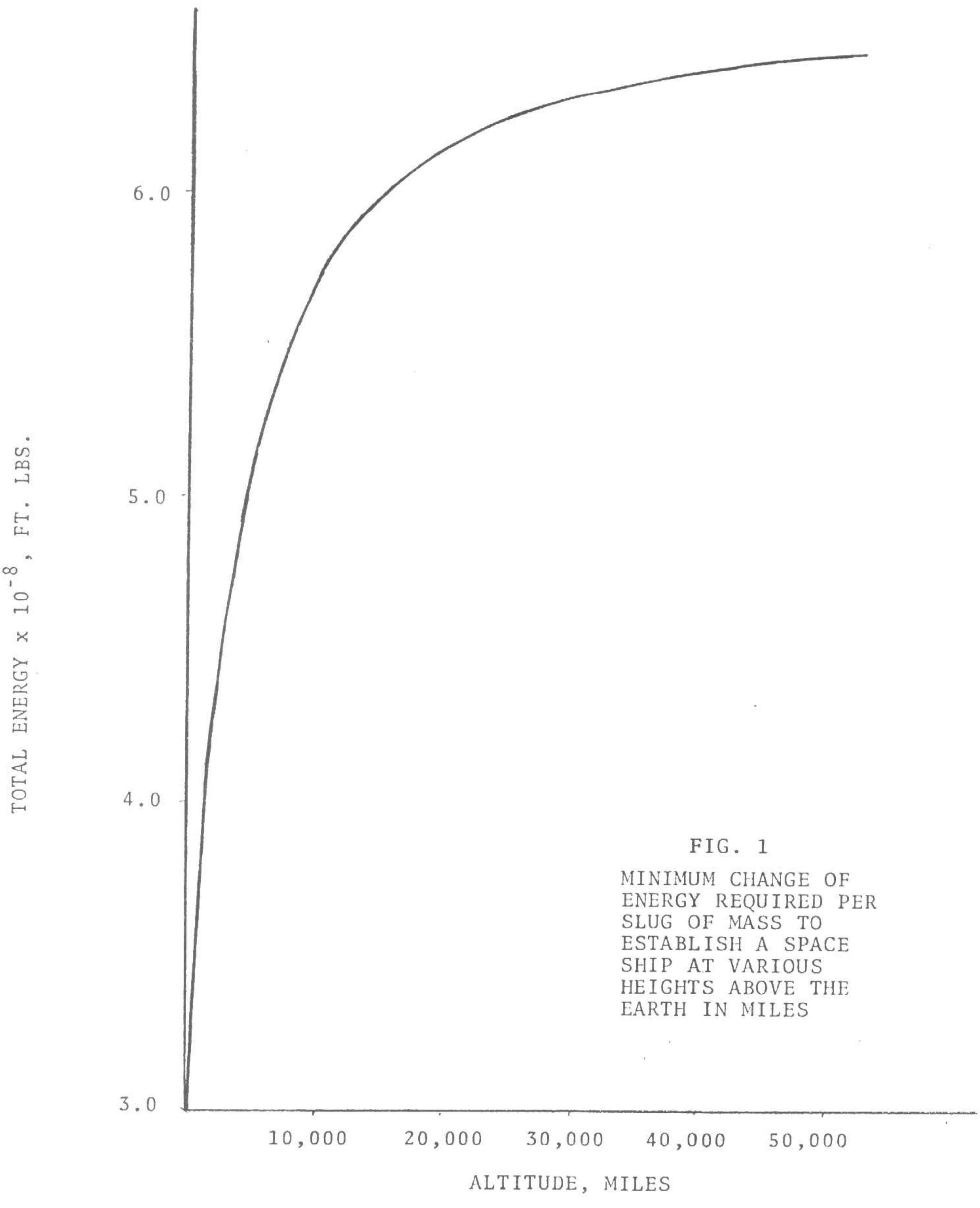


FIG. 1

MINIMUM CHANGE OF ENERGY REQUIRED PER SLUG OF MASS TO ESTABLISH A SPACE SHIP AT VARIOUS HEIGHTS ABOVE THE EARTH IN MILES

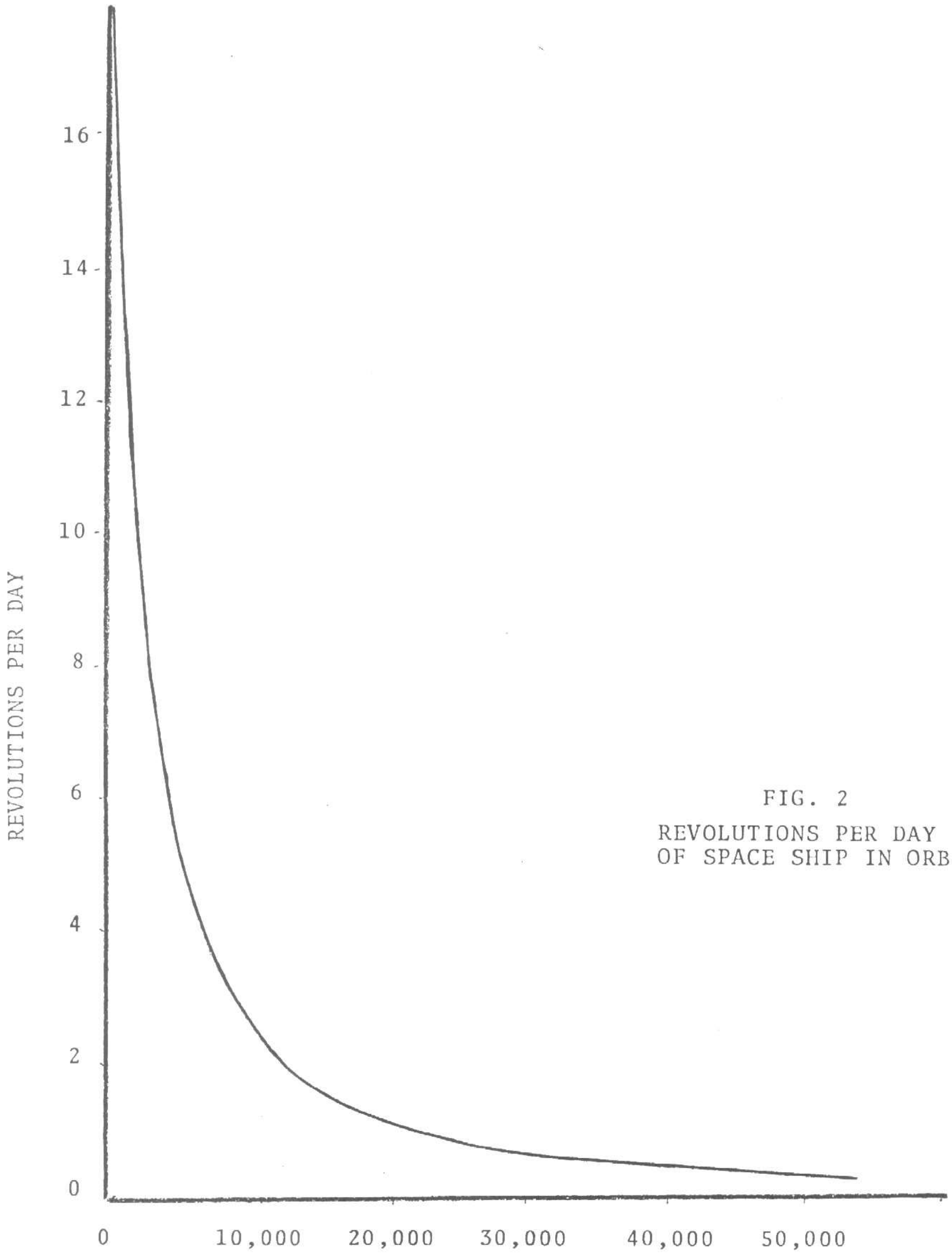


FIG. 2
REVOLUTIONS PER DAY
OF SPACE SHIP IN ORBIT

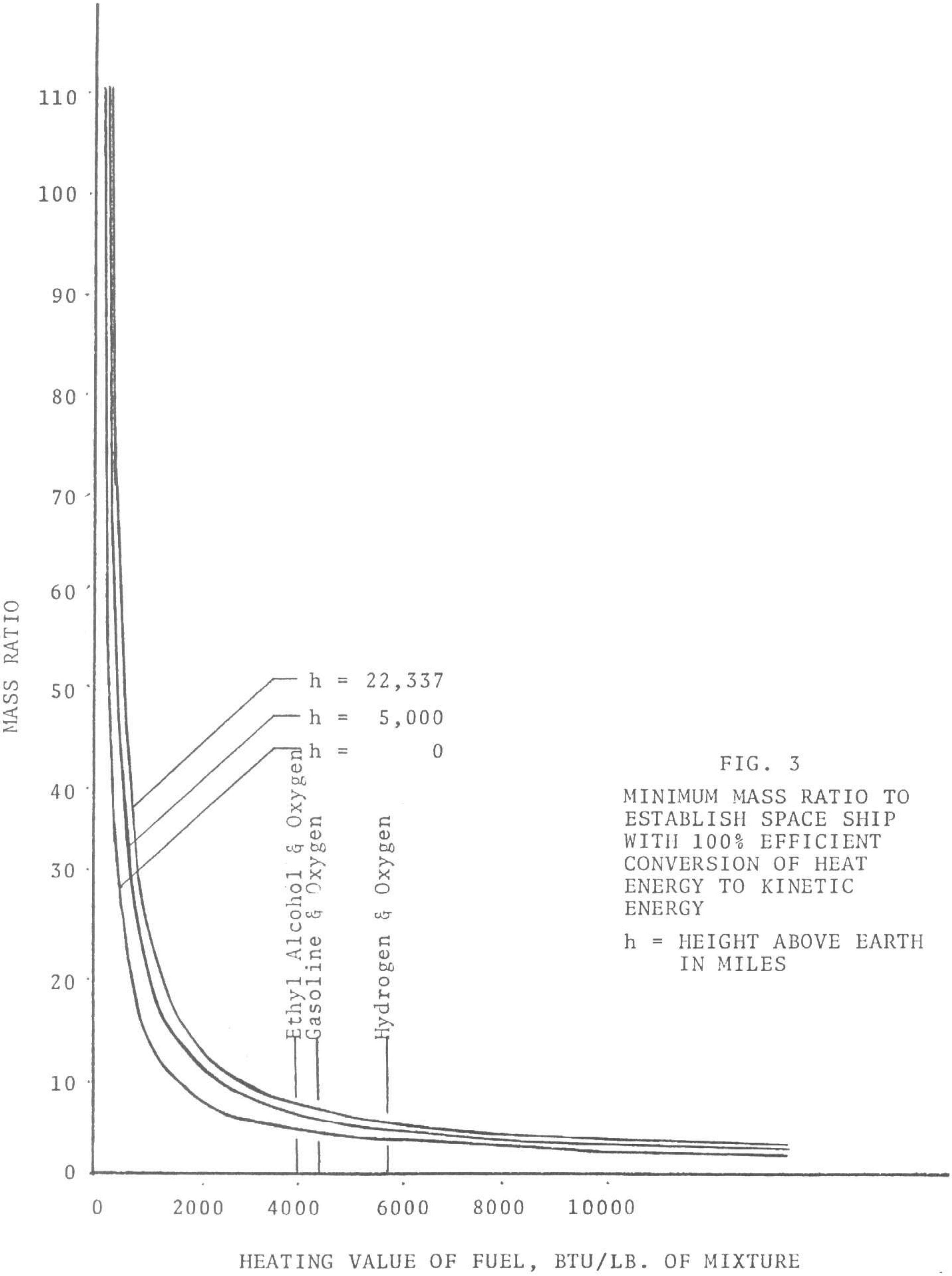


FIG. 3

MINIMUM MASS RATIO TO ESTABLISH SPACE SHIP WITH 100% EFFICIENT CONVERSION OF HEAT ENERGY TO KINETIC ENERGY

h = HEIGHT ABOVE EARTH IN MILES

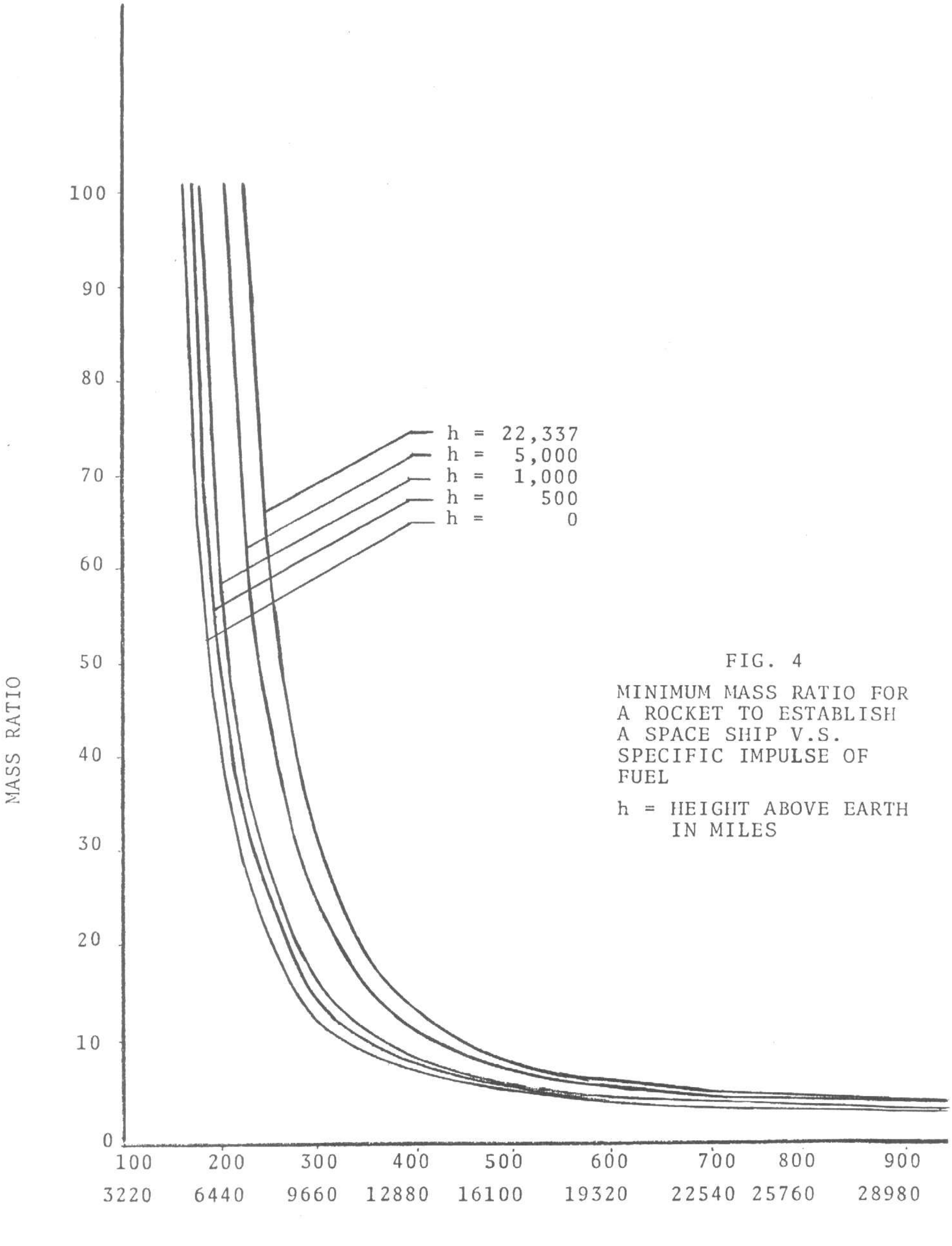


FIG. 4

MINIMUM MASS RATIO FOR A ROCKET TO ESTABLISH A SPACE SHIP V.S. SPECIFIC IMPULSE OF FUEL

h = HEIGHT ABOVE EARTH IN MILES

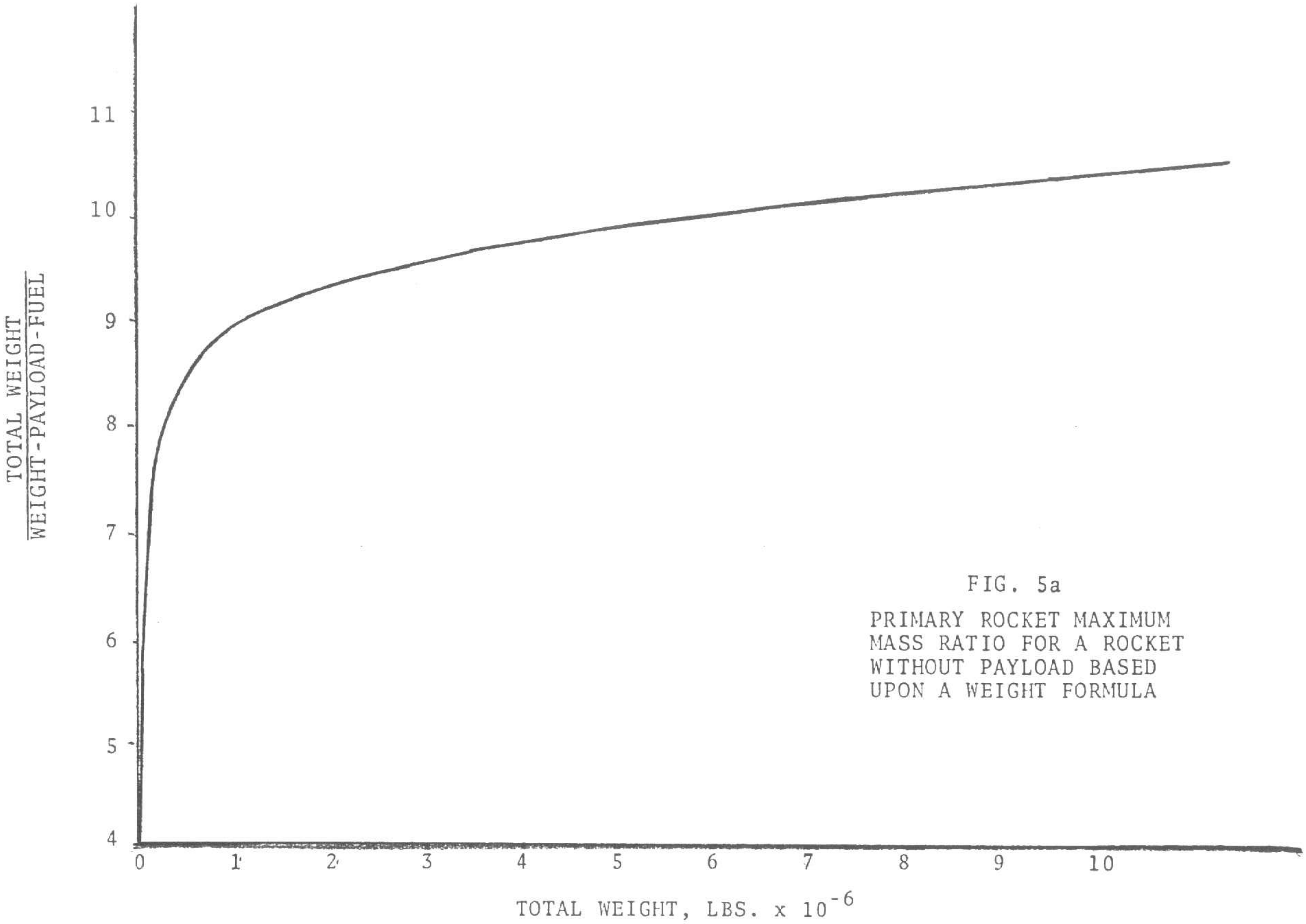


FIG. 5a

PRIMARY ROCKET MAXIMUM
MASS RATIO FOR A ROCKET
WITHOUT PAYLOAD BASED
UPON A WEIGHT FORMULA

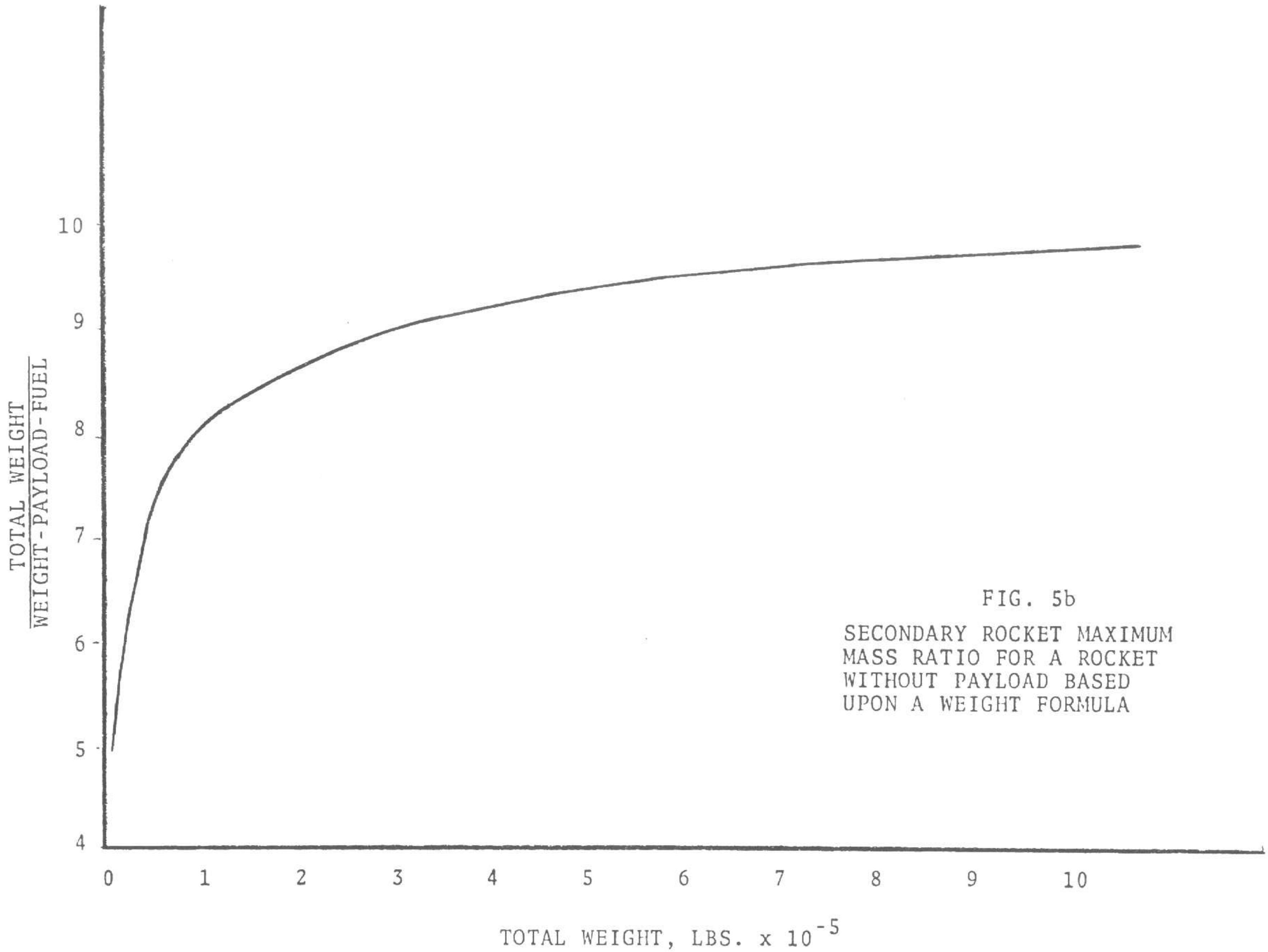


FIG. 5b

SECONDARY ROCKET MAXIMUM MASS RATIO FOR A ROCKET WITHOUT PAYLOAD BASED UPON A WEIGHT FORMULA

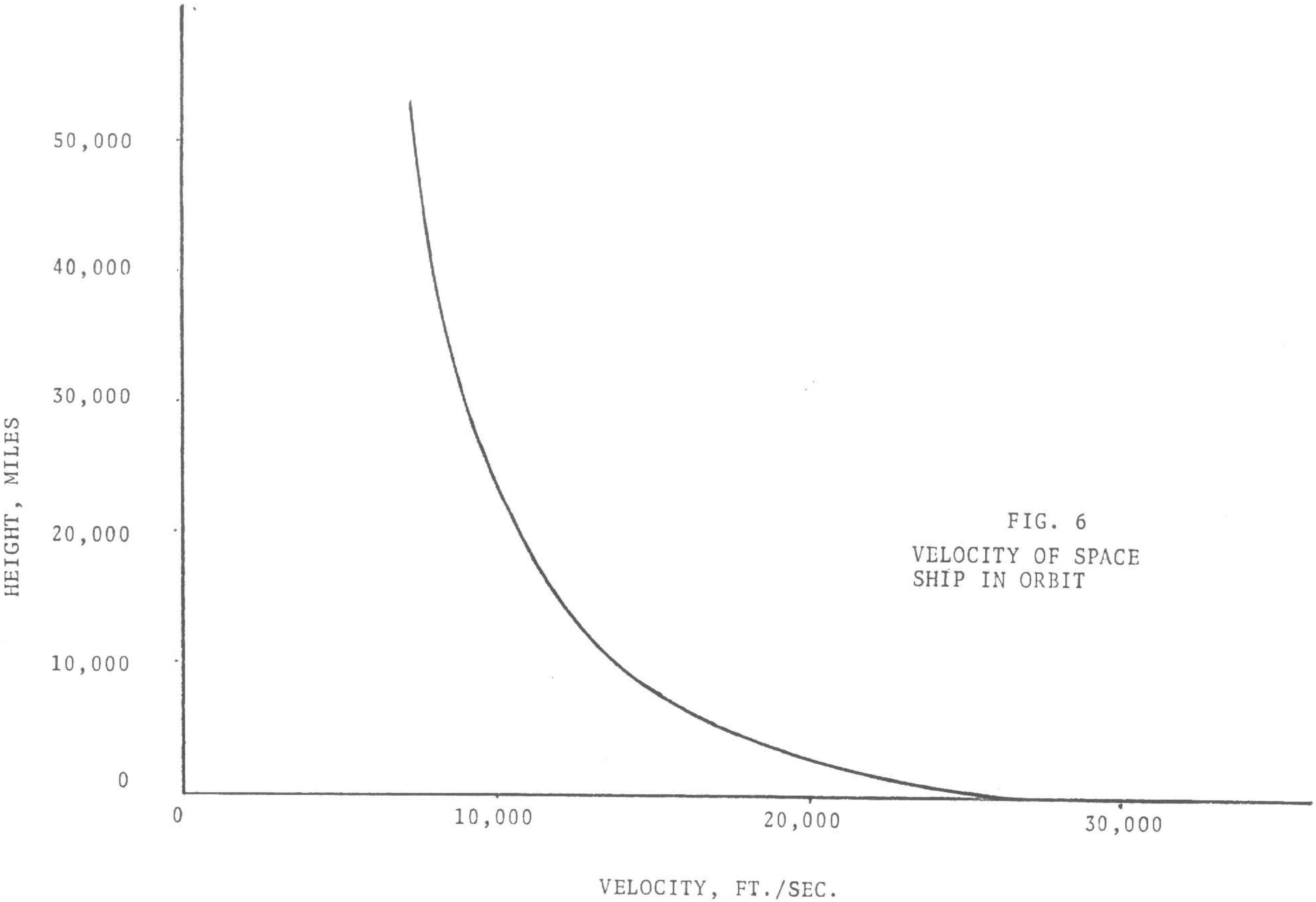


FIG. 6
VELOCITY OF SPACE
SHIP IN ORBIT

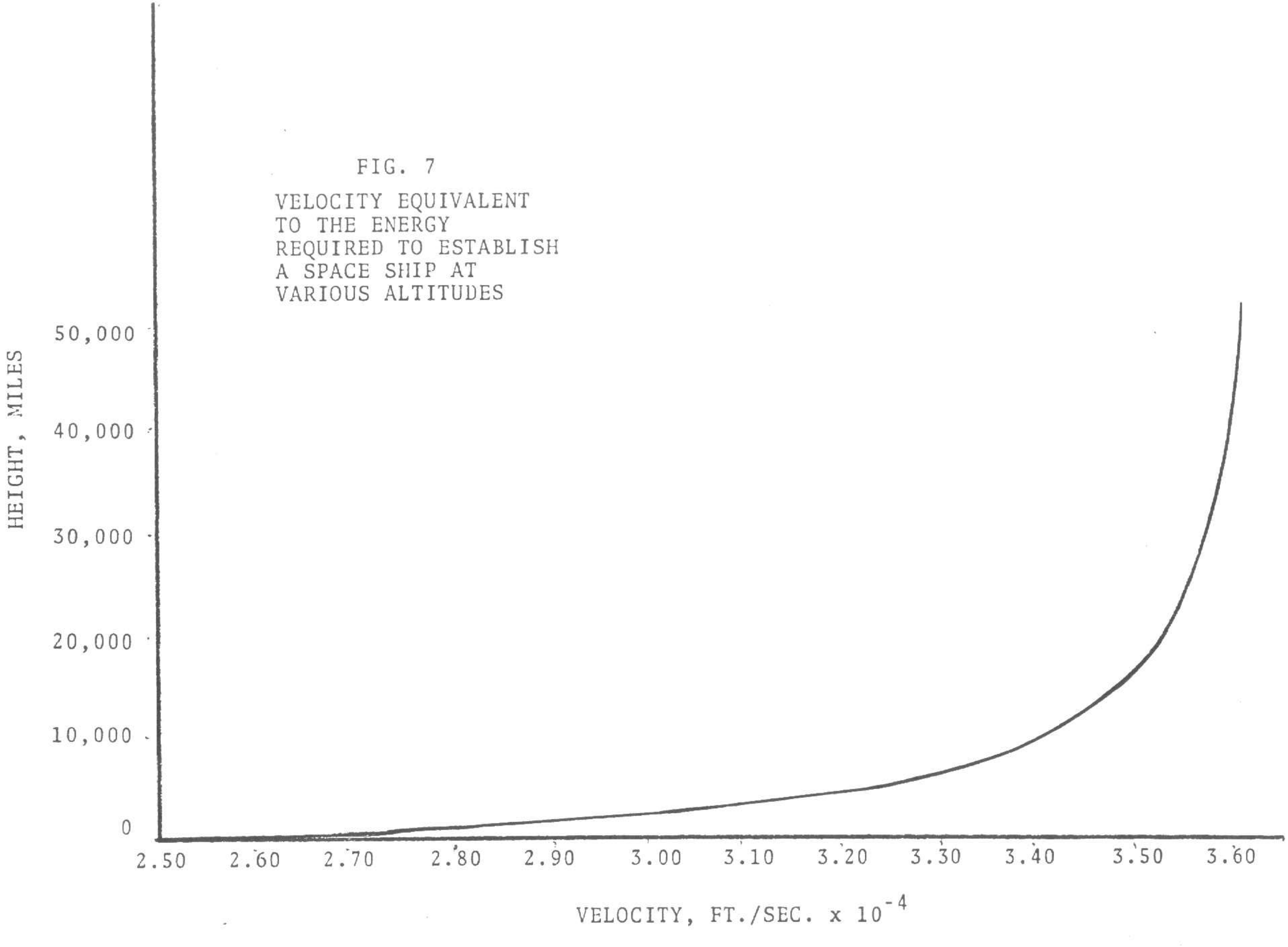


FIG. 7

VELOCITY EQUIVALENT TO THE ENERGY REQUIRED TO ESTABLISH A SPACE SHIP AT VARIOUS ALTITUDES

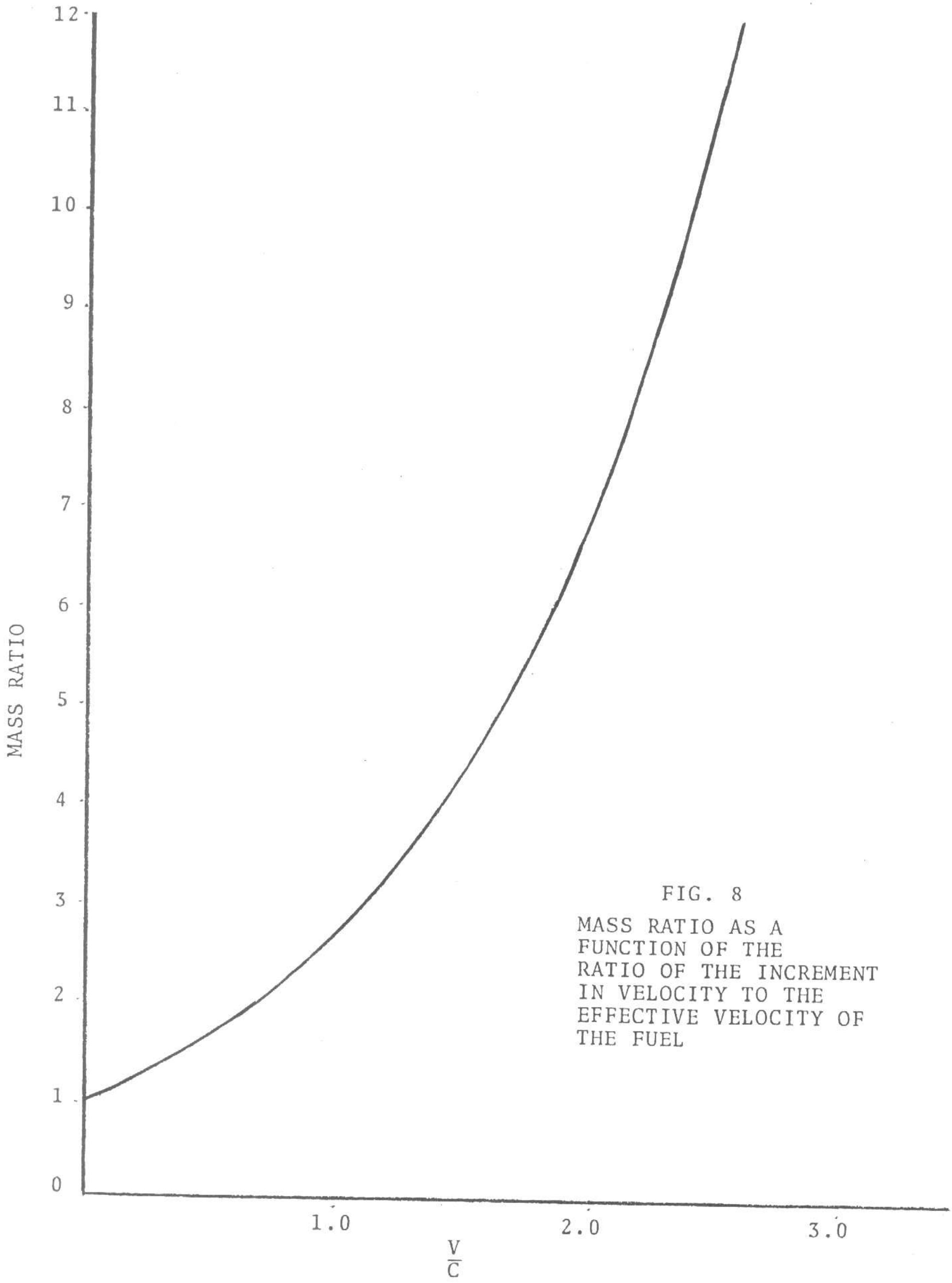


FIG. 8

MASS RATIO AS A FUNCTION OF THE RATIO OF THE INCREMENT IN VELOCITY TO THE EFFECTIVE VELOCITY OF THE FUEL

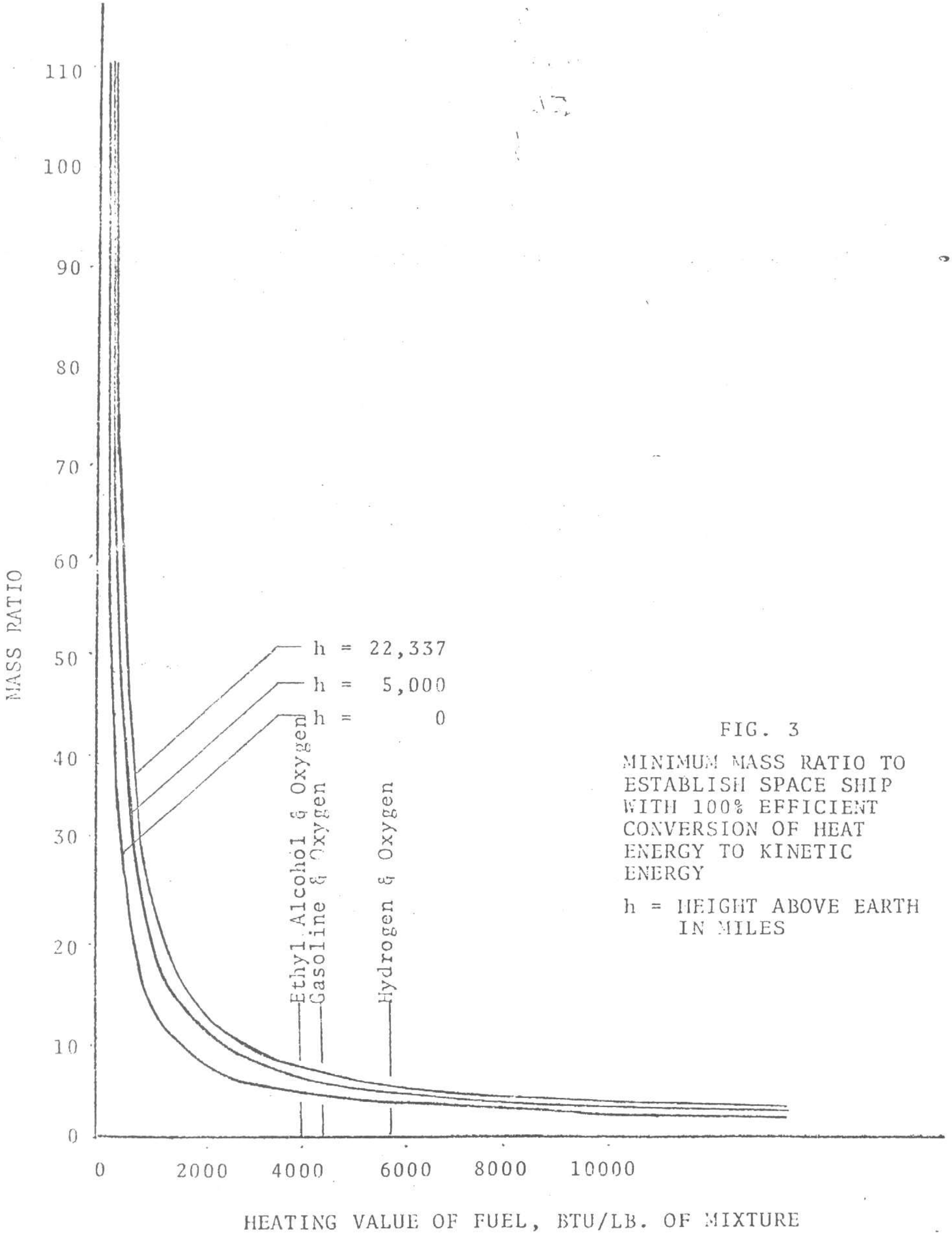


FIG. 3

MINIMUM MASS RATIO TO ESTABLISH SPACE SHIP WITH 100% EFFICIENT CONVERSION OF HEAT ENERGY TO KINETIC ENERGY

h = HEIGHT ABOVE EARTH IN MILES

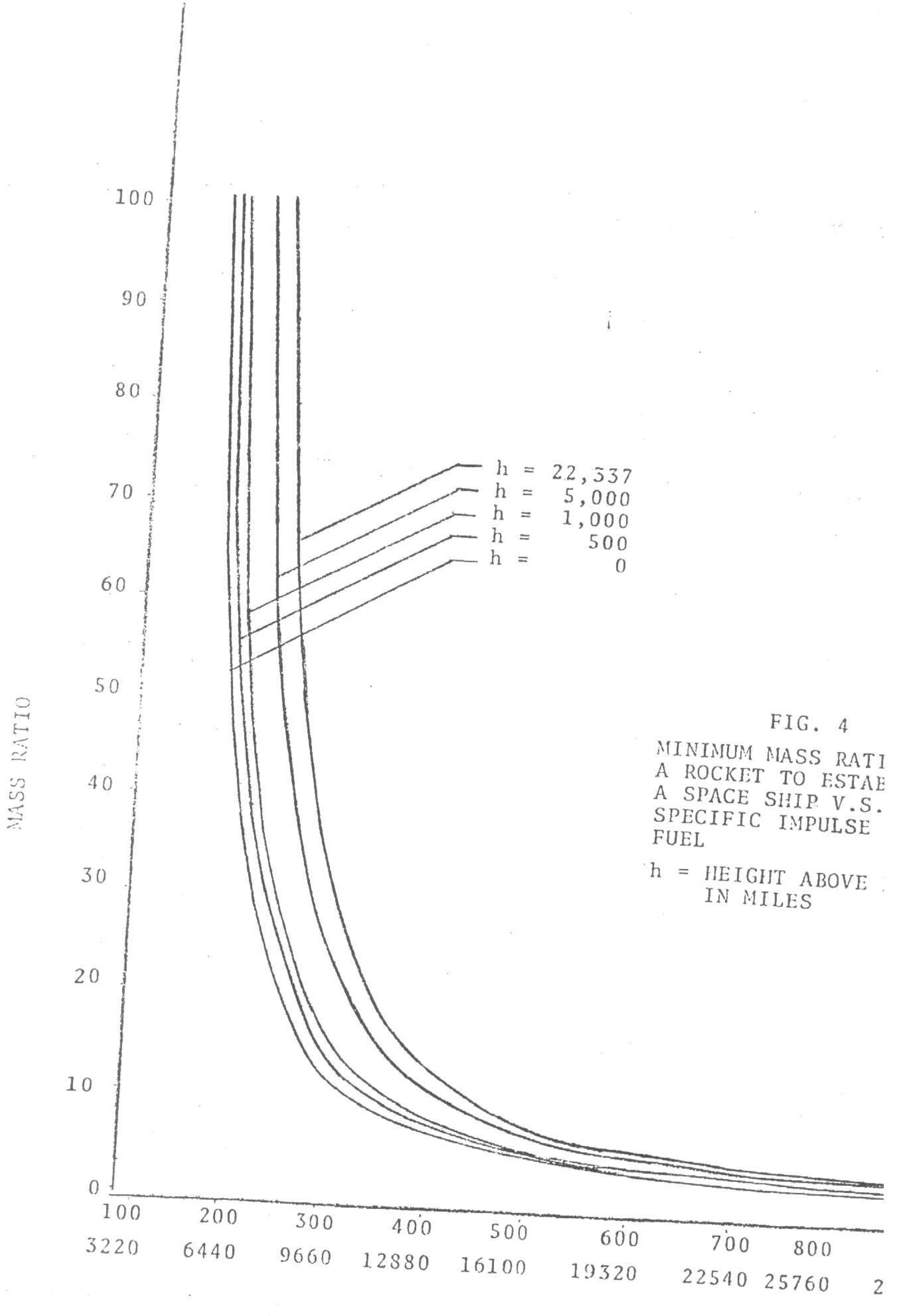


FIG. 4

MINIMUM MASS RATI
A ROCKET TO ESTAB
A SPACE SHIP V.S.
SPECIFIC IMPULSE
FUEL

h = HEIGHT ABOVE
IN MILES

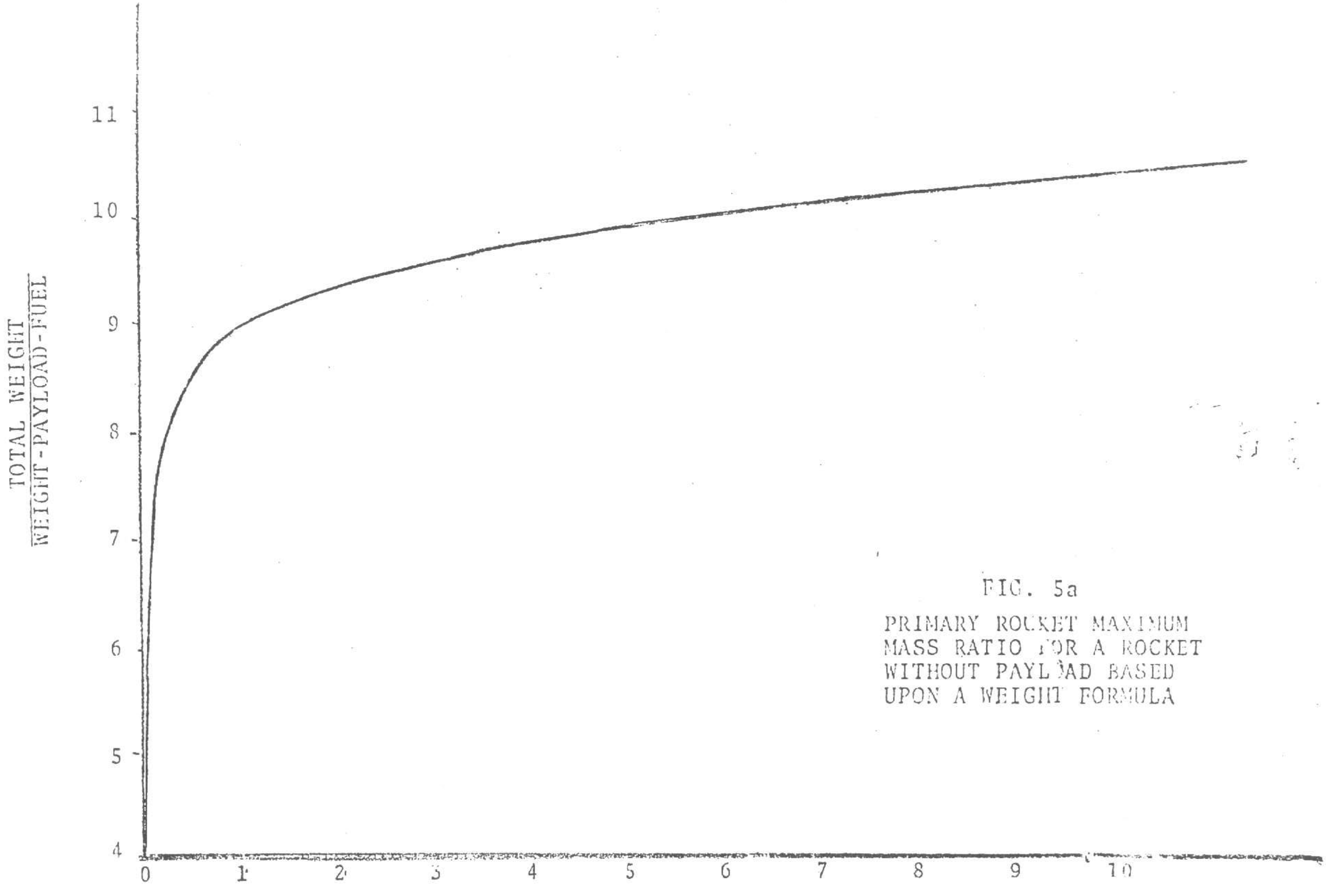


FIG. 5a

PRIMARY ROCKET MAXIMUM MASS RATIO FOR A ROCKET WITHOUT PAYLOAD BASED UPON A WEIGHT FORMULA

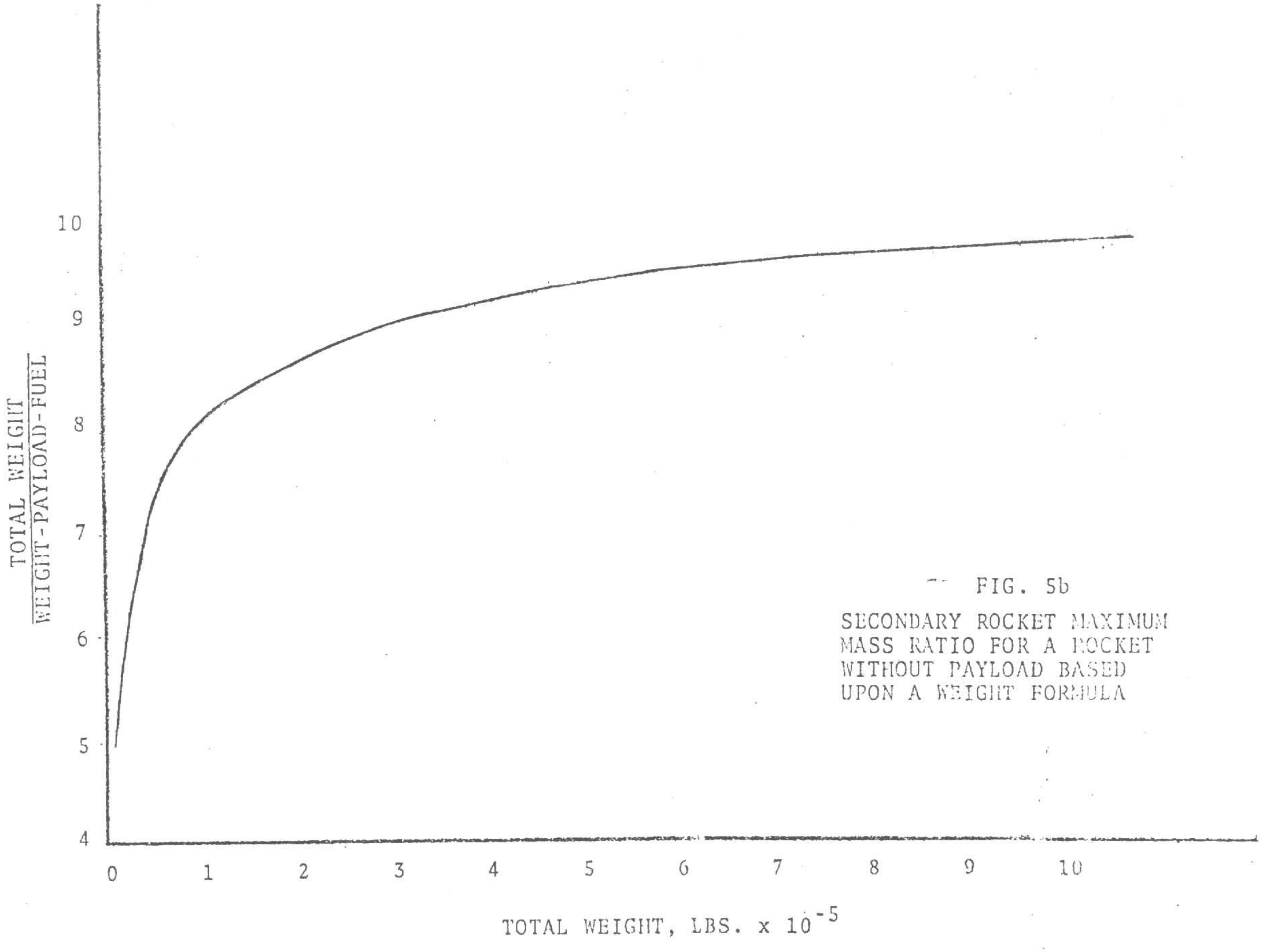


FIG. 5b

SECONDARY ROCKET MAXIMUM MASS RATIO FOR A ROCKET WITHOUT PAYLOAD BASED UPON A WEIGHT FORMULA

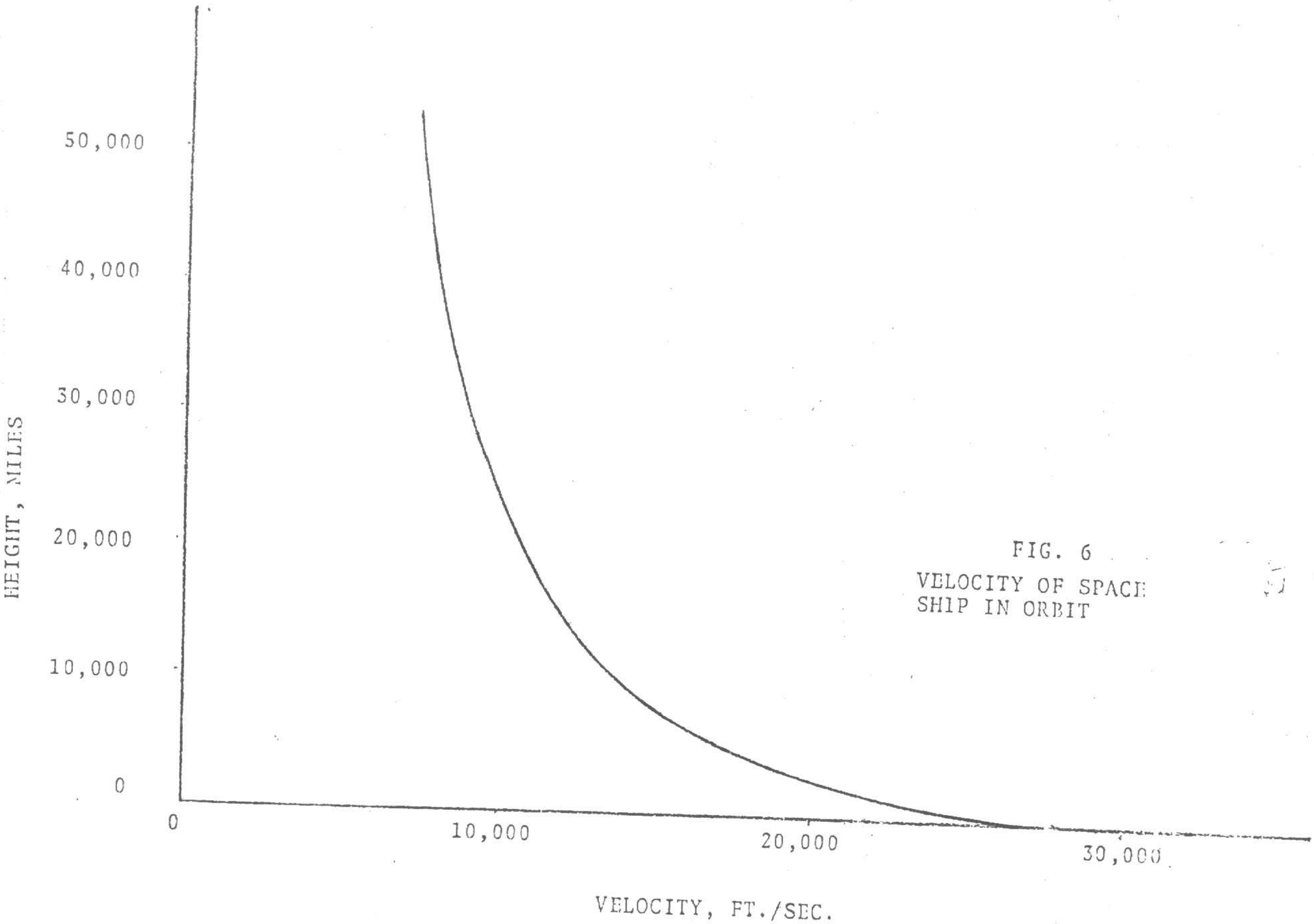


FIG. 6
VELOCITY OF SPACE SHIP IN ORBIT

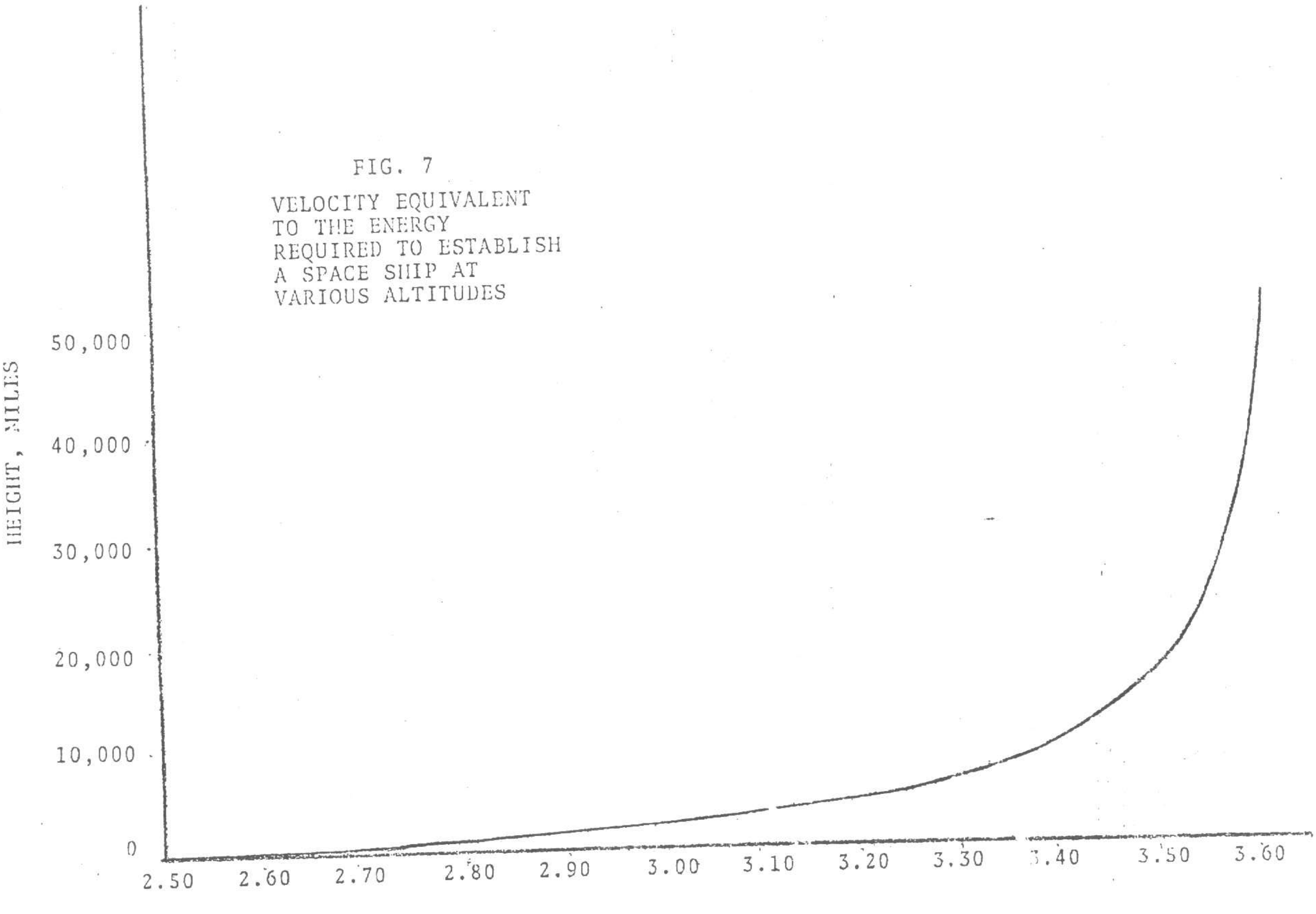


FIG. 7

VELOCITY EQUIVALENT TO THE ENERGY REQUIRED TO ESTABLISH A SPACE SHIP AT VARIOUS ALTITUDES

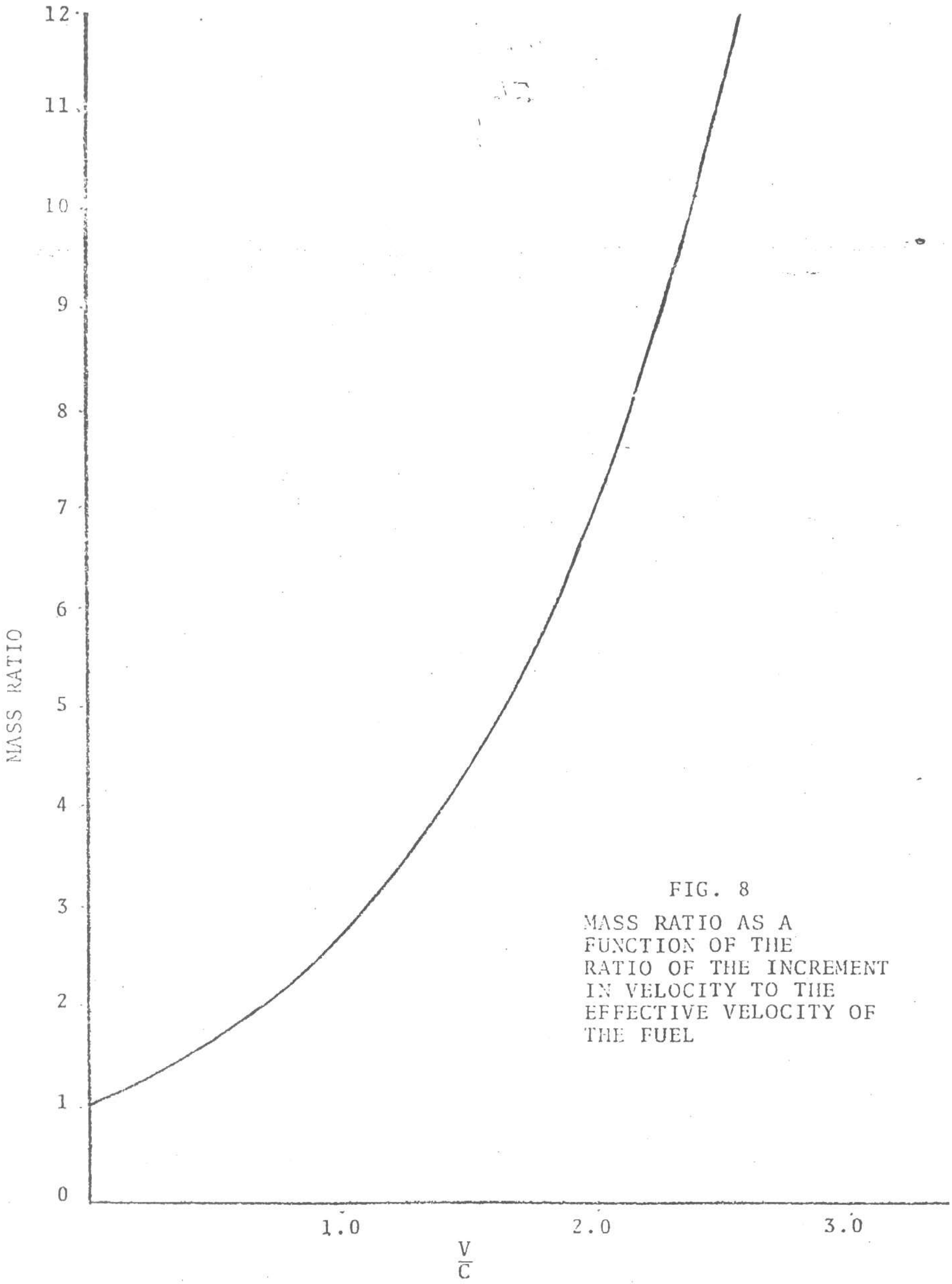


FIG. 8

MASS RATIO AS A FUNCTION OF THE RATIO OF THE INCREMENT IN VELOCITY TO THE EFFECTIVE VELOCITY OF THE FUEL